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A MATHEMATICAL MODEL FOR CONTROL OF OPTIMAL GROWTH OF BACTERIA

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ABSTRACT

In the present paper, a Mathematical model has developed for finding the control of infection disease by growth of bacterial population by assuming the mortality depends on the growth of bacterial population and the concentration of toxic substances by bacteria is discussed with the help of integro-differential equation.

KEYWORDS: Population, Bacteria, Mortality, Toxic Product

Bacteria, generally the smallest living organisms in the atmosphere grow in stages and are able to multiply exponentially, even in small areas over a period of hours or days. Bacterial growth is defined as the division one bacteria into two cells, taking place is called binary fission. Bacteria are responsible for some of the most fatal disease such as bubonic plague, food poisoning and meningitis have killed millions peoples. There is also a strong evidence that microbes may contribute to many non- infectious chronic diseases are caused by different types of micro-organisms. Infectious disease microbe that cause the disease immune system. The study of bacterial population the model is great interest in population dynamics because they play an important role in our environment and they have important industrial applications such as in fermentation technology, food environments also [Baranyi and Tamplin, 2004]. In advancement of medicinal-tecno society with there are many developing country having many part where child mortality rate is very high due to infection disease generated by bacteria [Keenan *et al.,* 2018]. There are different mathematical models has given by different mathematician to find the control of infection disease by bacteria time to time [Lawley, 2010] [Murray, 2002].

The simplest growth model for population growth of plants or animal species by Malthus [Malthus, 1978] is

$$\frac{dP}{dt} = bP, t > 0.$$

Where $P(t)$ is the number of individuals at time $t,$ and $b > 0$ is the specific growth rate of bacterial population.

This is the appropriate model exists for unlimited environment only. This model indicates $\mathrm{P} \to \infty$ as $\mathrm{t} \to \infty$. The population density first increases, but at higher density the rate of increase decreases giving a maximum limit of population. Pearl [Pearl, 1925] modified the above population model as

$$\frac{dP}{dt} = bP - dP^2, t > 0$$

Where $d > 0$ reflects the degree in which the population growth rate is reduced due to density increases. Different models for population growth for bacteria are discussed by Kapur 1985, Dabes 1973, Baranyi 1998 for dependence of its growth with concentration. Sharma *et.al.* 2014 studied a Mathematical model for n-species competition for aphid population in limited resources under toxic effects. They assumed that toxic substances produced by aphid become a limiting factor to their further growth by increasing the mortality rate to the deterioration of the environment by honeydew excretions of aphids. In the present paper we shall develop a population growth model of bacteria under certain assumptions that mortality depends on increasing density of bacteria colony as well as decrease in the number of organism by production of toxic substances by bacteria.

FORMULATION OF MODEL AND ITS SOLUTION

Consider $P(t)$ is total number of individuals at a time $t.$ The rate of change in the total number is equal to differences of birth and mortality rate which is affected by two factors one is by increasing density of bacterial population and second is by increasing concentration of toxic substance produced by bacteria, and covered area, the population growth model of bacteria can be given as:

$$\frac{\mathrm{dP}}{\mathrm{dt}} = \mathrm{bP} - \mathrm{kP}\int_0^{\mathrm{t}} \{\mathrm{P}_1(\mathrm{s}) + \mathrm{P}_2(\mathrm{s}) + \cdots + \mathrm{P}_n(\mathrm{s})\}\mathrm{ds} \quad \ldots(1)$$

Where $b > 0$ is specific growth rate of population and k is a positive constant suppose the toxic substances is produced by bacteria at a constant rate.

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This is an integro –differential equation which can be solved for $P(t)$ with initial conditions

or $\frac{dP}{dt} = bP - knP\int_0^t P(s)dS$ …(2)

where $P_1(s) + P_2(s) + \cdots + P_n(s) = n\,P(s)$

To solve above equation putting $x(t) = \int_0^t P(s)ds$

Equation (2) reduces as

$x'' = bx' - knxx'$

or $x'' = bx' - k'xx'$, …(3)

where $kn = k'$ is a positive constant.

Multiplying eq(3) both sides by e^{ax}, we get-

$$\frac{d}{dt}(e^{ax}x'') = \frac{d}{dt}(bx' - k'xx')e^{ax}$$

on integrating it between limit 0 to t we get-

$$e^{ax}x'' - P(0) = e^{ax}\left(\frac{b}{a} - \frac{k'}{a}x\right) - \left(\frac{b}{a}\right)$$

Here $x(0) = 0$ and $x'(0) = P(0)$ gives,

$$P(t) = P(0)e^{-ax} + \left(\frac{b}{a} - \frac{k'}{a}x\right) - \frac{b}{a}e^{-ax}$$

or $P(t) = \left(\frac{b}{a} - \frac{k'}{a}x\right) - \frac{b}{a}e^{-ax}$ …(4)

=f(x) say

or $\frac{dx}{dt} = f'(x)$

or $t = \int_0^x \frac{dx}{f(x)}$

equation (4) represents the bacterial populations at time t.

RESULTS AND DISCUSSION

On analyzing the nature of curve $P(t)$ obtaining $\frac{dP}{dt}$ and finding maximum value of $P(t)$.From equation(4) we have

$$\frac{dP}{dt} = f'(x)\frac{dx}{dt} = f'(x)P(t)$$

$$\frac{dP}{dt} = \left\{\frac{-k'}{a} - \left(\frac{b}{a} - p(0)\right)e^{-ax}(-a)\right\}k'\left\{\left(\frac{b}{a} - \frac{k'}{a}x\right) - \left(\frac{b}{a} - P(0)\right)e^{-ax}\right\}$$

$$\frac{dP}{dt} = \left\{\left(P(0) - \frac{b}{a}\right)(-ae^{-ax}) - \frac{k'}{a}\right\}\left\{\left(\frac{b}{a} - \frac{k'}{a}x\right) - \left(\frac{b}{a} - P(0)\right)e^{-bx}\right\} \quad ….(5)$$

There are following results:

(i) If $P(0) \geq \frac{b}{a}$ then $\frac{dP}{dt}$ is always negative and the bacterial population steadlly decreases to zero.

(ii) If $P(0) < \frac{b}{a}$ then $\left(\frac{dP}{dt}\right)_{t=0} = \left(\frac{dP}{dt}\right)_{x=0} = -a\left(P(0) - \frac{b}{a}\right) > 0$

Therefore the bacterial population increases at start and reaches a maximum when $\frac{dP}{dt} = 0$ which gives-

$$x = \frac{1}{a}log\left\{\frac{a(b-aP(0))}{k'}\right\}$$

From equation (4)

$$P_{max} = \frac{b}{a} - \frac{k'}{a^2}log\left\{\frac{a(b-aP(0))}{k'}\right\} < \frac{b}{a}$$

After this the bacterial population decreases and increases towards zero.

So we conclude that in above cases $(0) \geq \frac{b}{a}$ *and* $P(0) < \frac{b}{a}$, the bacterial populations tends to zero that is the bacterial population tends to extinction, and many disease due to infection will have to be control in this medicinal-techno society.

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